General Physics

THE LORENZ WATERWHEEL REVISITED

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The systematic study of chaos generally traces its origins back to Lorenz's numerical studies of atmospheric convection in the early 1960's, which led to such fundamental concepts as extreme sensitivity to initial conditions (the famous "butterfly effect") and the motion of trajectories in phase space around strange attractors. Further investigation of Lorenz's equations revealed that the dynamics of other nonlinear systems could be modeled by those equations, and researchers quickly looked for examples of such systems, especially simple mechanical analogs. Probably the best known contraption of this sort was invented by Willem Malkus and Lou Howard at MIT in the early 1970's. It consisted of a tilted wheel having hollow cylindrical cells around its circumference. Water was fed into the cells at the top of the tilted plane and leaked out through small holes in the bottom of each cell. Frictional torque was imposed with a viscous brake. Within certain regions of parameter space (input flow rate, magnitude of frictional torque, tilt angle, etc.), the wheel's angular velocity undergoes modulation and reversals, and the irregular timing of the reversals is chaotic. The Lorenz waterwheel is a common example of chaos in a mechanical system and is described or at least mentioned in almost every archive of information concerning non-linear dynamics and chaos. The Lorenz equations have been extensively studied both theoretically and numerically, although the extremely rich and complex structure of that system makes it a fruitful area for continuing research.

It is somewhat surprising, then, to note that no experimental studies of the Lorenz waterwheel have ever been published. No one doubts that it actually works – Malkus's original model and other subsequent versions provide adequate demonstrations of the effect – but the lack of published data from a systematic experimental study leaves a significant void in the literature and ignores a potentially valuable proving ground for experimental study of timely topics such as chaos synchronization.

In this talk, we will provide a theoretical description of the chaotic waterwheel leading to a system of equations that can be solved numerically. Analysis of the simulations allows us to estimate various parameters that will be used in the design of our apparatus. These parameters will be chosen so that the final design can exhibit as many different classes of motion (asymptotic, periodic, chaotic, etc.) as possible.